1 - 5 Components and length
Find the components of the vector v with initial point P and terminal point Q. Find |**v**|. Sketch |**v**|. Find the unit vector **u** in the direction of **v**.

1. P : (1, 1, 0), Q : (6, 2, 0)

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {1, 1, 0}; qQ = {6, 2, 0};
vec = qQ - pP
```

```
{5, 1, 0}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

$\sqrt{26}$

Below: find normalized version of vector.

```
euc2 = Normalize[vec]
```

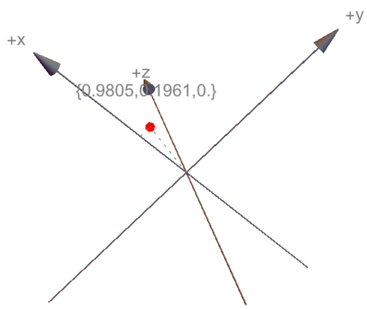
$\left\{\frac{5}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 0\right\}$

The green cells above match the answers in the text.

Below: find normalized version in decimal form.

```
euc3 = N[Normalize[vec], 4]
{0.9806, 0.1961, 0}
```

```
pad = N[euc3 + {.3, .3, .3}, 4]
{1.28058, 0.496116, 0.3}
```



3. P : (-3.0, 4.0, -0.5), Q : (5.5, 0, 1.2)

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {-3.0, 4.0, -0.5}; qQ = {5.5, 0, 1.2};
vec = qQ - pP
```

```
{8.5, -4., 1.7}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
9.54673
```

```
PossibleZeroQ[Chop[euc1] - Chop[√91.14]]
True
```

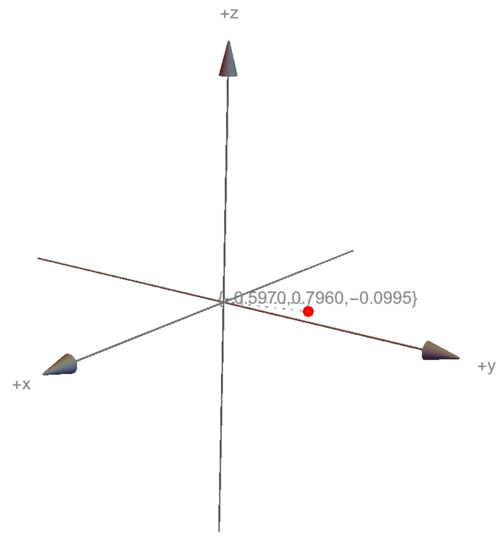
Below: find normalized version of vector.

```
euc2 = N[Normalize[vec], 4]
```

```
{0.890357, -0.418992, 0.178071}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
{1.19036, -0.118992, 0.478071}
```

The green cells above match the answers in the text.



5. P : (0, 0, 0), Q : (2, 1, -2)

```
ClearAll["Global`*"]
```

Below: reposition vector tail to origin.

```
pP = {0, 0, 0}; qQ = {2, 1, -2};
vec = qQ - pP
```

```
{2, 1, -2}
```

Below: calculate length of vector.

```
euc1 = Norm[vec]
```

```
3
```

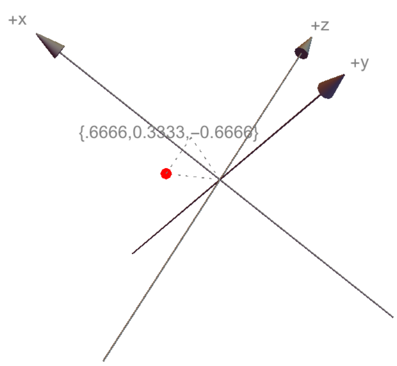
Below: find normalized version of vector.

```
euc2 = N[Chop[Normalize[vec], 10^-4]]
```

```
{0.666667, 0.333333, -0.666667}
```

```
pad = N[euc2 + {.3, .3, .3}, 4]
{0.966667, 0.633333, -0.366667}
```

The green cells above match the answers in the text (no entry for vector length).



6 - 10 Find the terminal point Q of the vector **v** with components as given and initial point P. Find $|\mathbf{v}|$.

7. $\frac{1}{2}$, 3, $-\frac{1}{4}$; P : $\left(\frac{7}{2}, -3, \frac{3}{4}\right)$

```
ClearAll["Global`*"]
```

```
vec = {1/2, 3, -1/4}; po = {7/2, -3, 3/4};
que = vec + po
```

$\{4, 0, \frac{1}{2}\}$

```
el = Norm[vec + po]
```

$\frac{\sqrt{65}}{2}$

```
FullSimplify[√65/2 == √16.25]
```

True

The green cells above match the answers in the text.

9. 6, 1, -4; P : (-6, -1, -4)

```
ClearAll["Global`*"]
```

```
vec = {6, 1, -4}; po = {-6, -1, -4};
que = vec + po
```

{0, 0, -8}

```
e1 = Norm[que]
```

8

The green cells above match the answers in the text.

11 - 18 Addition, scalar multiplication

Let a = {3, 2, 0} = 3 i + 2 j; b = {-4, 6, 0} = 4 i + 6 j; c = {5, -1, 8} = 5 i - j + 8 k, d = {0, 0, 4} = 4 k

Find

11. 2 a, $\frac{1}{2}$ a, -a

```
ClearAll["Global`*"]
```

```
aa = {3, 2, 0}; bb = {-4, 6, 0}; cc = {5, -1, 8}; dd = {0, 0, 4}
```

{0, 0, 4}

```
2 aa
```

{6, 4, 0}

```
1/2 aa
```

$\{\frac{3}{2}, 1, 0\}$

```
-aa
```

```
{-3, -2, 0}
```

The green cells above match the answers in the text.

13. b + c, c + b

```
bb + cc
```

```
{1, 5, 8}
```

```
cc + bb
```

```
{1, 5, 8}
```

15. 7 (c - b), 7 c - 7 b

```
7 (cc + bb)
```

```
{7, 35, 56}
```

```
7 (cc - bb)
```

```
{63, -49, 56}
```

17. (7 - 3) a, 7 a - 3 a

```
(7 - 3) aa
```

```
{12, 8, 0}
```

```
7 aa - 3 aa
```

```
{12, 8, 0}
```

The green cells above match the answers in the text.

21 - 25 Forces, resultant

Find the resultant in terms of components and its magnitude.

21. p = {2, 3, 0}, q = {0, 6, 1}, u = {2, 0, -4}

```
res = {2 + 0 + 2, 3 + 6 + 0, 0 + 1 - 4}
```

```
{4, 9, -3}
```

Norm[res]

$\sqrt{106}$

matt = $\begin{pmatrix} 2 & 3 & 0 \\ 0 & 6 & 1 \\ 2 & 0 & -4 \end{pmatrix}$

{{2, 3, 0}, {0, 6, 1}, {2, 0, -4}}

e2 = RowReduce[matt]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

23. u = {18, -1, 0}, v = $\left\{\frac{1}{2}, 0, \frac{4}{3}\right\}$, w = $\left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$

res = $\left\{8 + \frac{1}{2} - \frac{17}{2}, -1 + 0 + 1, 0 + \frac{4}{3} + \frac{11}{3}\right\}$

{0, 0, 5}

Norm[res]

5

mat = $\begin{pmatrix} 8 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{4}{3} \\ -\frac{17}{2} & 1 & \frac{11}{3} \end{pmatrix}$

$\left\{\{8, -1, 0\}, \left\{\frac{1}{2}, 0, \frac{4}{3}\right\}, \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}\right\}$

e1 = RowReduce[mat]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Above: the reduced echelon form says that the three vectors are linearly independent. Therefore there is no possibility of finding factors which express one in terms of the other two.

25. u = {3, 1, -6}, v = {0, 2, 5}, w = {3, -1, -13}

res = {3 + 0 + 3, 1 + 2 - 1, -6 + 5 - 13}

{6, 2, -14}

```
Norm[res]
```

$2\sqrt{59}$

$$\text{mattt} = \begin{pmatrix} 3 & 1 & -6 \\ 0 & 2 & 5 \\ 3 & -1 & -13 \end{pmatrix}$$

```
{{3, 1, -6}, {0, 2, 5}, {3, -1, -13}}
```

```
RowReduce[mattt]
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

Above: for this one, the text says the result is 2**u.** I don't think that is a correct statement. In fact, the reduced echelon form says that the three vectors are linearly independent. Therefore is it not doubtful that I can find factors which express one in terms of the other two?

26 - 37 Forces, velocities

27. Find **p** such that **u, v, w** in problem 23 and **p** are in equilibrium.

From problem 37, it is understood that vectors, considered as forces, are in equilibrium when they form a 'force polygon.' This polygon will have to be 4-sided. From the s.m., I find that ". . . "Equilibrium" means that the resultant of the given forces is the zero vector." Meaning I need to find **p** such that **u** + **v** + **w** + **p** = 0.

```
ClearAll["Global`*"]
```

$$uu = \{8, -1, 0\};\ vv = \left\{\frac{1}{2}, 0, \frac{4}{3}\right\};\ ww = \left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$$

$$\left\{-\frac{17}{2}, 1, \frac{11}{3}\right\}$$

```
pp = - (uu + vv + ww)
```

```
{0, 0, -5}
```

29. Restricted resultant. Find all **v** such that the resultant of **v, p, q, u** with **p, q, u** as in problem 21 is parallel to the *xy*-plane.

The resultant vector in problem 21 was {4, 9, −3}. So any vector of the form {*x*, *y*, −3} will be parallel to the xy-plane when added to the resultant of problem 21.

31. For what *k* is the resultant of {2, 0, -7}, {1, 2, -3}, and {0, 3, *k*} parallel to the *xy*-plane?

```
res1 = {2 + 1, 0 + 2, -3 - 7}
{3, 2, -10}
```

```
k = 10
```

```
10
```

The green cells above match the answers in the text.

32. If |**p**| = 6 and |**q**| = 4, what can you say about the magnitude and direction of the resultant? Can you think of an application to robotics?

33. Same question as in problem 32 if |**p**| = 9, |**q**| = 6, |**u**| = 3.

Comprising three cloud spheres. Any resulting magnitude greater than or equal to zero and less than or equal to 18. Any octant.

34. Relative velocity. If airplanes *A* and *B* are moving southwest with speeds $|\boldsymbol{v}_A| = 550$ mph, and northwest with speed $|\boldsymbol{v}_B| = 450$ mph, respectively, what is the relative velocity $\mathbf{v} = \boldsymbol{v}_B - \boldsymbol{v}_A$ of *B* with respect to *A*?

```
ClearAll["Global`*"]
```

```
N[Solve[2 aa^2 == 550^2], 4]
{{aa → -388.9}, {aa → 388.9}}
```

```
aavec = {-388.9, -388.9}
{-388.9, -388.9}
```

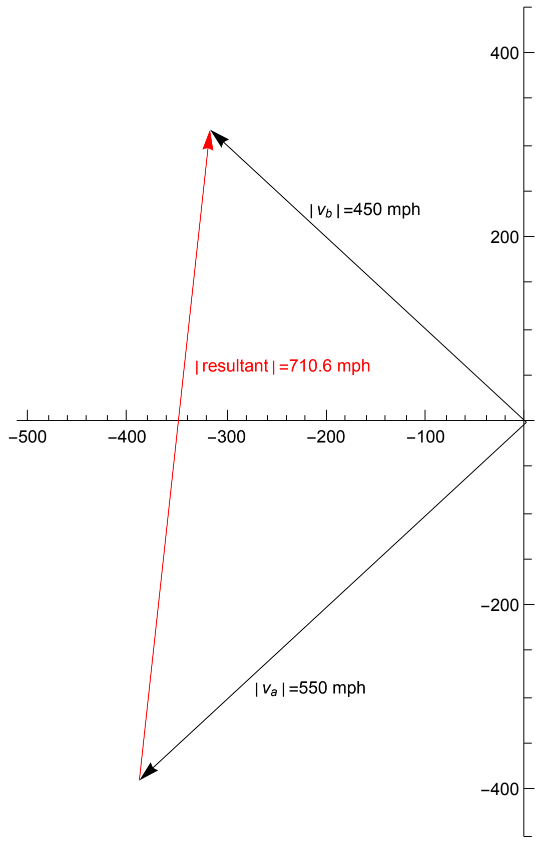
```
N[Solve[2 bb^2 == 450^2], 4]
{{bb → -318.2}, {bb → 318.2}}
```

```
bbvec = {-318.2, 318.2}
{-318.2, 318.2}
```

```
bbvec - aavec
{70.7, 707.1}
```

```
Norm[bbvec - aavec]
710.626
```

The relative velocity of bbvec with respect to aavec is 70.7 mph east, 707.1 mph north.



35. Same question as in problem 34 for two ships moving northeast with speed $|\boldsymbol{v}_A| = 22$ knots and west with speed $|\boldsymbol{v}_B| = 19$ knots.

```
ClearAll["Global`*"]

N[Solve[2 as² == 22²], 4]
{{as → -15.56}, {as → 15.56}}

asvec = {15.56, 15.56}
{15.56, 15.56}

N[Solve[ bs² == 19²], 4]
{{bs → -19.00}, {bs → 19.00}}

bsvec = {-19, 0}
{-19, 0}
```

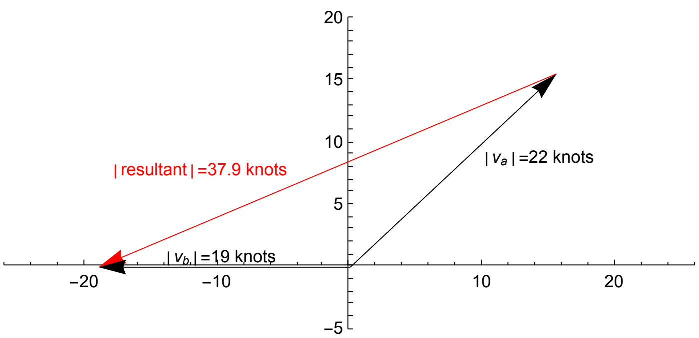
The problem doesn't say what the perspective is, let's say I want velocity of ship b with respect to ship a.

```
resulta = bsvec - asvec
```

```
{-34.56, -15.56}
```

The relative velocity of ship b with respect to ship a is 34.6 knots west, 15.6 knots south.

```
Norm[resulta]
37.9013
```

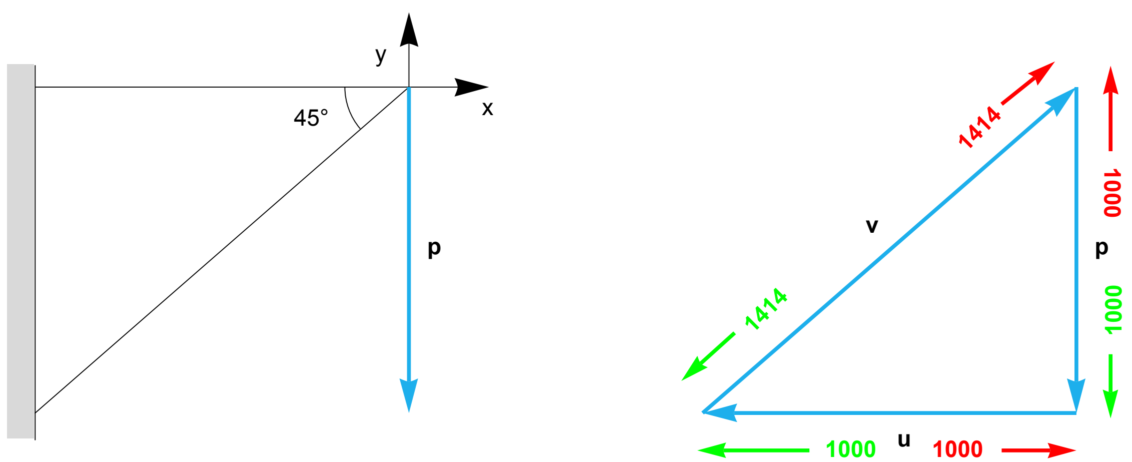



```
N[{-19 - 22/√2, -22/√2}]
{-34.5563, -15.5563}
```

The green cells above match the answers in the text.

37. Force polygon. Truss. Find the forces in the system of two rods (truss) in the figure, where $|\mathbf{p}| = 1000$ nt. *Hint.* Forces in equilibrium form a polygon, the *force polygon.*

Out[222]=



Truss *Force polygon*

```
ClearAll["Global`*"]
```

This looks like a simple problem that could be taken care of in a line or two. There is the older approach set out at *https://rip94550.wordpress.com/2012/03/19/trusses-example-1/*. This blogsite on applied math is directed/written/curated by Rip and appears to use *Mathemat-*

ica 8. The details still appear to work okay. However, the problem differs slightly from the current one.

There is also the YouTube video at *https://www.youtube.com/watch?v=H60Q70FZI9M*, uploaded by MrClean. It has forces directed in the same sense as the present problem. I applied analogous forces to the force polygon above, green for imposed force, red for reaction force. All forces cancel, meaning the truss remains stationary. For some reason the text answer does not mention the forces in **v**. Maybe a force polygon does not work the same as the method of joints.